

UNCLASSIFIED

AD 401 742

*Reproduced
by the*

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

FTD-TT- 63-157

63-32

401742

TRANSLATION

THE MAGNETIC FIELD, EXCITED BY THE PRIMARY
WINDING, IN THE AIR GAP OF AN INDUCTION
MACHINE WITH A CURVED STATOR

By

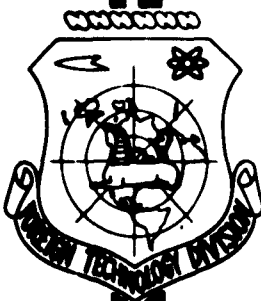
A. A. Lebedev

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE

OHIO



401742

FOOTIA

AD 110.

UNEDITED ROUGH DRAFT TRANSLATION

THE MAGNETIC FIELD, EXCITED BY THE PRIMARY
WINDING, IN THE AIR GAP OF AN INDUCTION
MACHINE WITH A CURVED STATOR

By: A. A. Lebedev

English Pages: 15

Source: Russian Periodical, Electromekhanika,
No. 5, 1959, pp. 14-23.

T-15
SOV/144-59-0-5-3/14

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION SERVICES BRANCH
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

THE MAGNETIC FIELD, EXCITED BY THE PRIMARY WINDING,
IN THE AIR GAP OF AN INDUCTION MACHINE WITH A
CURVED STATOR

A. A. Lebedev

The use of an ordinary a-c machine winding in induction motors with curved stators leads to undesirable effects in the operation of the machine which result in the sudden display of an edge effect.

The edge effect results from the interruption of the magnetic circuit of the stator and gives rise to pulsating magnetic fields at the ends of the arcs of the curved stator. These fields have an effect on the magnitude of magnetic induction in the air gap throughout the entire length of the arc. In addition to losses in the rotor and stator due to the pulsating magnetic fields and the braking moments associated with them, the edge effect causes asymmetry of the linear currents and consequently, additional braking moments. The edge effect shows up less in the operation of machines with a large number of poles than in those with a small number of poles.

A new type of curved-stator winding is proposed in order to reduce the influence of the edge effect on the operation of an induction machine. The new winding differs from those in use in that the linear

load at the ends of the arcs of the curved stator are reduced to zero.

The application of a winding of the new type makes it possible to raise the efficiency of a motor with a curved stator and a massive rotor. The power factor is also improved and the asymmetry of the linear currents is practically eliminated [2].

The work of Prof. G. I. Shturman [1] is devoted to the question associated with the conditions of excitation of magnetic fields with the interruption of the magnetic circuit of the stator. Formulas derived in this work cannot be used for the evaluation of magnetic fields in the air gap of an induction machine having the new type of stator winding. In addition, these formulas are not altogether accurate.

This article is devoted to an evaluation of magnetic fields excited in the air gap by a stator having the new type of winding. We will solve this problem on the basis of the following assumptions:

1. The steel of the stator and rotor is not saturated and has a relative permeability $\mu = \text{const.}$
2. The reluctance of the steel of the stator and rotor in a radial direction is allowed for by a corresponding increase of the air gap.
3. We will consider the fundamental wave of magnetomotive force.
4. We will neglect leakage currents.
5. We will consider that there is no current in the rotor.
6. We will consider the reluctances throughout the circuit to be the same for the yokes of the stator and rotor.

The last assumption seems at first glance gross, but considering that the magnetic saturation of the steel of the stator and rotor is assumed to be approximately the same, the difference in reluctances of

the yoke of the rotor and stator will be determined only by the difference of the average lengths of the lines of induction in them. We take into account this insignificant difference (which decreases with an increase in the diameter of the rotor) by the fact that we maintain the general length of the lines of induction in the yoke of the rotor and stator, taking the length of the sections of the by-pass circuit in the yoke equal to the length of the arc along the air gap.

Disregard of the reluctances of the steel of the rotor and stator leads to a qualitatively different picture of the magnetic field in the air gap.

In order to evaluate the magnetic shunting of the air gap by those parts of the steel of the rotor and stator which do not have a winding, we assume the length of arc of the steel to be greater, by an amount Y , than the length of the winding at each end.

Depending on the character of the variation of the linear load along the length of arc of the curved stator, we divide the latter into five parts (Fig. 1):

1. section without winding,
2. linear load rises from zero to a nominal value,
3. linear load remains constant,
4. linear load decreases from nominal to zero, and
5. section without winding.

We make the origin of the cylindrical coordinates coincide with the axis of rotation of the rotor. The origin for reading the angle φ coincides with the middle of the arc of the curved stator. The positive direction of φ is taken to be counterclockwise. We assume the radius of the rotor to be equal to unity; therefore we can consider the length of arc to be equal to the angle. In the final results the

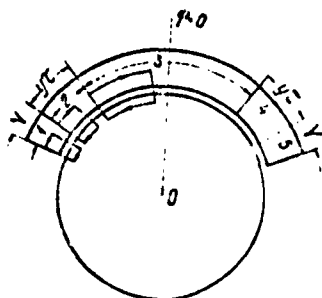


Fig. 1. Magnetic circuit of an induction machine with curved stator.

length of arc is substituted for the angles. The positive direction of induction in the rotor and stator is indicated by the arrows in Fig. 1. The direction from the rotor to the stator is taken as the positive direction of induction. The positive direction of current in the stator winding is assumed to be the direction from the observer into the plane of the drawing (Fig. 1).

The instantaneous value of the total current along the length of arc of the curved stator is determined from the traveling-wave equation

$$i = A \cos \left(\omega t - \frac{\pi}{\tau} \varphi \right), \quad (1)$$

where i = instantaneous value of current,
 A = amplitude value of the linear load,
 τ = polar division, and
 ω = angular frequency of the power-supply current.

Let us consider the case when the linear load at the ends drops to zero according to a sine law.

By virtue of the symmetry of the magnetic field with respect to the middle of the arc of the curved stator, we will consider the first three sections.

In the first section, the instantaneous current value is zero.

For the second section, the instantaneous current value may be determined as follows

$$i = A \cos \frac{\pi(\varphi - \varphi_{2H})}{2(\varphi_{2H} - \varphi_{1H})} \cos \left(\omega t - \frac{\pi}{\tau} \varphi \right), \quad (2)$$

where φ_{2H} is the angle corresponding to the start of the second section and

φ_{2K} is the angle corresponding to the end of the second section.

For the third section, the instantaneous current value is determined from Formula (1).

Under the conditions assumed, the law of total current for the by-pass circuit in the first section may be written in the following form

$$B_{1H} - B_1 \delta + \frac{2}{\mu_0 h} \int_{\varphi_{1H}}^{\varphi_1} B_{R1} d\varphi = 0, \quad (3)$$

where B_{1H} = induction in the air gap at the start of the first section,

B_1 = induction in the air gap in the first section,

μ_0 = permeability of air

δ = air gap with regard for the radial reluctance of the steel of the stator rotor,

φ_{1H} = angle corresponding to the start of the first section,

φ_1 = present value of the angle in the first section, and

B_{R1} = induction in the yoke of stator and rotor in the first section.

Induction in the yoke in the first section may be expressed in terms of the induction in the air gap:

$$B_{R1} = \frac{1}{h} \int_{\varphi_{1H}}^{\varphi_1} B_1 d\varphi, \quad (4)$$

where h is the height of the yoke of the stator. Substituting (4) into (3) we obtain

$$B_{1H} - B_1 + k^2 \int_{\varphi_{1H}}^{\varphi_1} B_1 d\varphi = 0, \quad (5)$$

where

$$k^2 = \frac{2}{\mu_0 h \delta}. \quad (6)$$

We differentiate expression (5) twice with respect to φ_1

$$\frac{d^2 B_1}{d\varphi^2} - k^2 B_1 = 0. \quad (7)$$

The solution of this equation has the following form

$$B_1 = C_1 e^{k\varphi} + C_2 e^{-k\varphi}. \quad (8)$$

For the by-pass circuit in the second section, the law of total current is written in the form

$$\frac{B_{2n} - B_2}{\mu_0} \delta + \frac{2}{\mu_0 \delta} \int_{\varphi_{1n}}^{\varphi_2} B_{R1} d\varphi = \int_{\varphi_{1n}}^{\varphi_2} A \cos m(\varphi - \varphi_{1n}) \cos(\omega t - n\varphi) d\varphi, \quad (9)$$

where

$$m = \frac{\pi}{2(\varphi_{1n} - \varphi_{1n})},$$

$$n = \frac{\pi}{\delta}.$$

The angle and induction subscripts here and in the following formulas are the same as in Formulas (2) and (3) and correspond to the section number.

Induction in the yoke in the second section may be expressed in terms of induction in the air gap thus

$$B_{R2} = \frac{1}{h} \int_{\varphi_{1n}}^{\varphi_{1n}} B_1 d\varphi + \frac{1}{h} \int_{\varphi_{2n}}^{\varphi_2} B_2 d\varphi, \quad (10)$$

then in place of (9) we obtain

$$B_{1n} - B_2 + k^2 \int_{\varphi_{1n}}^{\varphi_{1n}} \int_{\varphi_{2n}}^{\varphi_2} B_1 d\varphi^2 + k^2 \int_{\varphi_{2n}}^{\varphi_2} \int_{\varphi_{1n}}^{\varphi_{1n}} B_2 d\varphi^2 =$$

$$= \frac{\mu_0 A}{\delta} \int_{\varphi_{2n}}^{\varphi_2} \cos m(\varphi - \varphi_{1n}) \cos(\omega t - n\varphi) d\varphi. \quad (11)$$

Differentiating twice with respect to φ_2 we obtain

$$\frac{d^2 B_2}{d\varphi^2} - k^2 B_2 = -\frac{\mu_0 A}{\delta} [n \cos m(\varphi - \varphi_{1n}) \sin(\omega t - n\varphi) -$$

$$- m \sin m(\varphi - \varphi_{1n}) \cos(\omega t - n\varphi)]. \quad (12)$$

Solution of this equation takes the following form

$$B_2 = \frac{\mu_0 A}{2\delta} \left[\frac{m+n}{k^2 + (m+n)^2} \sin[\omega t - (m+n)\varphi + m\varphi_{1n}] - \right.$$

$$\left. - \frac{m-n}{k^2 + (m-n)^2} \sin[\omega t + (m-n)\varphi - m\varphi_{1n}] \right] + C_3 e^{k\varphi} + C_4 e^{-k\varphi}. \quad (13)$$

For the circuit in the third section, the law of total current is written in the following manner

$$\frac{B_{3n} - B_3}{\mu_0} \delta + \frac{2}{\mu_0 \mu_0} \int_{\varphi_{3n}}^{\varphi_3} B_{R3} d\varphi = \int_{\varphi_{3n}}^{\varphi_3} A \cos(\omega t - n\varphi) d\varphi. \quad (14)$$

Induction in the yoke in the third section is expressed in terms of induction in the air gap thus

$$B_{R3} = \frac{1}{h} \int_{\varphi_{1n}}^{\varphi_{1n}'} B_1 d\varphi + \frac{1}{h} \int_{\varphi_{2n}}^{\varphi_{2n}'} B_2 d\varphi + \frac{1}{h} \int_{\varphi_{3n}}^{\varphi_{3n}'} B_3 d\varphi, \quad (15)$$

then in place of (14) we obtain

$$\begin{aligned} B_{3n} - B_3 + k^2 \int_{\varphi_{3n}}^{\varphi_{1n}'} \int_{\varphi_{1n}}^{\varphi_{1n}'} B_1 d\varphi^2 + k^2 \int_{\varphi_{3n}}^{\varphi_{2n}'} \int_{\varphi_{2n}}^{\varphi_{2n}'} B_2 d\varphi^2 + k^2 \int_{\varphi_{3n}}^{\varphi_{3n}'} \int_{\varphi_{3n}}^{\varphi_{3n}'} B_3 d\varphi^2 = \\ = \frac{\mu_0 A}{\delta} \int_{\varphi_{3n}}^{\varphi_3} \cos(\omega t - n\varphi) d\varphi. \end{aligned} \quad (16)$$

Differentiating twice with respect to φ_3 we obtain

$$\frac{d^2 B_3}{d\varphi^2} - k^2 B_3 = -\frac{\mu_0 A n}{\delta} \sin(\omega t - n\varphi). \quad (17)$$

The solution of this equation takes the following form

$$B_3 = \frac{\mu_0 A n}{\delta(k^2 + n^2)} \sin(\omega t - n\varphi) + C_1 e^{k\varphi} + C_2 e^{-k\varphi}. \quad (18)$$

In order to determine the six constants of integration C_1, C_2, C_3, C_4, C_5 and C_6 , in addition to the three fundamental equations (3), (11) and (16) we use the continuity equation for the magnetic flux:

$$\operatorname{div} B = 0, \quad (19)$$

which gives the two additional equations

$$B_{1n} = B_{2n}, \quad (20)$$

$$B_{2n} = B_{3n}. \quad (21)$$

In order to obtain the sixth equation for the determination of the constants of integration we use the condition of symmetry of the magnetic field with respect to the center of arc of the curved stator. This condition makes it possible to assert that the derivative of the

envelope of the set of curves of magnetic induction obtained from expression (18) is equal to zero at the middle of the arc.

Taking the derivative of the envelope of the set of curves obtained from expression (18) for various moments of time, with respect to φ and setting φ equal to zero, after elementary transformations, we obtain

$$C_5 = C_6 = 0. \quad (22)$$

Thus, substituting (8), (13), and (18) into (3), (11), (16), (20), and (21), and taking into account (22), we obtain the following six equations for the determination of the constants of integration

$$(23)$$

$$\begin{aligned} C_1 e^{k_1 \tau_{1K}} + C_2 e^{-k_1 \tau_{1K}} &= 0; \\ C_1 e^{k_1 \tau_{1K}} + C_2 e^{-k_1 \tau_{1K}} + C_3 e^{k_2 \tau_{1K}} + C_4 e^{-k_2 \tau_{1K}} &= \\ \frac{Mk}{m+n} \cos \alpha + \frac{Nk}{m-n} \cos \beta. \end{aligned} \quad (24)$$

$$\begin{aligned} C_2 e^{k_1 \tau_{1K}} + C_4 e^{-k_1 \tau_{1K}} + C_3 e^{k_2 \tau_{1K}} + C_5 e^{-k_2 \tau_{1K}} &= \\ \left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{lk}{n} \right) \cos(\omega t - \pi \tau_{1K}). \end{aligned} \quad (25)$$

$$\begin{aligned} C_1 e^{k_1 \tau_{1K}} + C_2 e^{-k_1 \tau_{1K}} + C_3 e^{k_2 \tau_{1K}} + C_4 e^{-k_2 \tau_{1K}} &= M \sin \alpha + N \sin \beta; \\ C_3 e^{k_2 \tau_{1K}} + C_4 e^{-k_2 \tau_{1K}} - C_5 e^{k_2 \tau_{1K}} - C_6 e^{-k_2 \tau_{1K}} &= (N - M + \Pi) \sin(\omega t - \pi \tau_{1K}); \\ C_5 = C_6 &= 0. \end{aligned} \quad \begin{matrix} (26) \\ (27) \\ (28) \end{matrix}$$

where

$$\begin{aligned} M &= \frac{\mu_0 A}{2\delta} \cdot \frac{m+n}{k^2 + (m+n)^2}; \quad N = \frac{\mu_0 A}{2\delta} \cdot \frac{m-n}{k^2 + (m-n)^2}; \\ \Pi &= \frac{\mu_0 A n}{\delta(k^2 + n^2)}; \\ \alpha &= \omega t - (m+n)\tau_{2K} + m\tau_{1K}; \quad \beta = \omega t + (m-n)\tau_{2K} - m\tau_{1K}. \end{aligned}$$

In expressions (24) - (27) it is considered that $\varphi_{1K} = \varphi_{2H}$ and $\varphi_{2K} = \varphi_{2H}$. Simultaneous solution of Eqs. (23)-(28) gives

$$\begin{aligned} C_1 = & \left\{ \left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) \operatorname{ch} k p \tau \sin(\omega t - \pi p) - (M - N) \operatorname{sh} k p \tau \cos(\omega t - \pi p) + \right. \\ & \left. + \left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{\Pi k}{n} \right) \operatorname{ch} k \tau (p - y) \cos[\omega t - \pi(p - y)] + \right. \\ & \left. + (N - M + \Pi) \operatorname{sh} k \tau (p - y) \sin[\omega t - \pi(p - y)] \right\} \frac{e^{-k(p+y)}}{2 \operatorname{sh} k(p+y)}; \end{aligned} \quad (29)^*$$

* sh = sinh; ch = cosh

$$C_2 = \left\{ \left(\frac{N\kappa}{m-n} - \frac{M\kappa}{m+n} \right) ch\kappa p \tau \sin(\omega t - \pi p) - (M+N) sh\kappa p \tau \cos(\omega t - \pi p) + \right. \\ \left. + \left(\frac{M\kappa}{m+n} + \frac{N\kappa}{m-n} - \frac{11\kappa}{n} \right) ch\kappa \tau (p-y) \cos[\omega t - \pi(p-y)] + \right. \\ \left. + (N-M+11) sh\kappa \tau (p-y) \sin[\omega t - \pi(p-y)] \right\} \frac{e^{h(p\tau+Y)}}{2sh\kappa(p\tau+Y)}; \quad (30)$$

$$C_3 = \left[\left(\frac{N\kappa}{m-n} - \frac{M\kappa}{m+n} \right) ch\kappa Y \sin(\omega t - \pi p) + \right. \\ \left. + (M+N) sh\kappa Y \cos(\omega t - \pi p) \right] \frac{1}{2sh\kappa(p\tau+Y)} + \\ + \left\{ \left(\frac{M\kappa}{m+n} + \frac{N\kappa}{m-n} - \frac{11\kappa}{n} \right) ch\kappa \tau (p-y) \cos[\omega t - \pi(p-y)] + \right. \\ \left. + (N-M+11) sh\kappa \tau (p-y) \sin[\omega t - \pi(p-y)] \right\} \frac{e^{-h(p\tau+Y)}}{2sh\kappa(p\tau+Y)}; \quad (31)$$

$$C_4 = \left[\left(\frac{N\kappa}{m-n} - \frac{M\kappa}{m+n} \right) ch\kappa Y \sin(\omega t - \pi p) + (N + \right. \\ \left. + M) sh\kappa Y \cos(\omega t - \pi p) \right] \frac{1}{2sh\kappa(p\tau+Y)} + \\ + \left\{ \left(\frac{M\kappa}{m+n} + \frac{N\kappa}{m-n} - \frac{11\kappa}{n} \right) ch\kappa \tau (p-y) \cos[\omega t - \pi(p-y)] + \right. \\ \left. + (N-M+11) sh\kappa \tau (p-y) \sin[\omega t - \pi(p-y)] \right\} \frac{e^{h(p\tau+Y)}}{2sh\kappa(p\tau+Y)}; \quad (32)$$

$$C_5 = C_6 = \left\{ \left(\frac{N\kappa}{m-n} - \frac{M\kappa}{m+n} \right) ch\kappa Y \sin(\omega t - \pi p) + (M+N) sh\kappa Y \cos(\omega t - \pi p) + \right. \\ \left. + \left(\frac{M\kappa}{m+n} + \frac{N\kappa}{m-n} - \frac{11\kappa}{n} \right) ch\kappa (Y+y\tau) \cos[\omega t - \pi(p-y)] - \right. \\ \left. - (N-M+11) sh\kappa (Y+y\tau) \sin[\omega t - \pi(p-y)] \right\} \frac{1}{2sh\kappa(p\tau+Y)}; \quad (33)$$

where $Y = \varphi_{1H} - \varphi_{1K}$, the length of the block of steel of the stator projecting beyond the active zone

$\gamma\tau = \varphi_{2H} - \varphi_{2K}$, the length of the arc on which the variation of the linear load from zero to nominal takes place.

Substituting the values of C_1 , C_2 , C_3 , C_4 , C_5 , and C_6 into the expression for induction, we obtain

$$\begin{aligned}
B_1 = & (-1)^p \left\{ \left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) chkp\tau \sin \omega t + (M+N) shkp\tau \cos \omega t + \right. \\
& + (-1)^y \left[\left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{\Pi k}{n} \right) chk\tau (p-y) \cos \omega t + \right. \\
& \left. \left. + (N-M+11) shk\tau (p-y) \sin \omega t \right] \right\} \frac{chk(p\tau + Y - \bar{\varphi})}{shk(p\tau + Y)}; \quad (34)
\end{aligned}$$

$$\begin{aligned}
B_2 = & -M \cos \left[\omega t - (m+n)\bar{\varphi} + \frac{\pi p}{2y} \right] - N \cos \left[\omega t + (m-n)\bar{\varphi} - \frac{\pi p}{2y} \right] + \\
& + (-1)^p \left\{ \left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) chkY \sin \omega t + \right. \\
& \left. + (M+N) shkY \cos \omega t \right\} \frac{chk\bar{\varphi}}{shk(p\tau + Y)} + (-1)^{p-y} \left[\left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \right. \right. \\
& \left. \left. - \frac{\Pi k}{n} \right) chk\tau (p-y) \cos \omega t + \right. \\
& \left. + (N-M+11) shk\tau (p-y) \sin \omega t \right] \frac{chk(p\tau + Y - \bar{\varphi})}{shk(p\tau + Y)}; \quad (35)
\end{aligned}$$

$$\begin{aligned}
B_3 = & 11 \sin \left(\omega t - \frac{\pi}{2} \bar{\varphi} \right) + (-1)^p \left\{ \left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) chkY \sin \omega t + \right. \\
& \left. + (M+N) shkY \cos \omega t + (-1)^y \left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{\Pi k}{n} \right) chk\tau (Y+y\tau) \cos \omega t - \right. \\
& \left. + (-1)^y (N-M+11) shk(Y+y\tau) \sin \omega t \right\} \frac{chk\bar{\varphi}}{shk(p\tau + Y)}; \quad (36)
\end{aligned}$$

where $\bar{\varphi}$ is the present value of the coordinates over the length of arc of the stator. Or

$$\begin{aligned}
B_1 = & (-1)^p \left\{ \left[\left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) chkp\tau + \right. \right. \\
& \left. + (-1)^y (N-M+11) shk\tau (p-y) \right] \cdot \left[(-1)^y \left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{\Pi k}{n} \right) chk\tau (p-y) - \right. \\
& \left. \left. + (M+N) shkp\tau \right] \right\} \frac{chk(p\tau + Y - \bar{\varphi})}{shk(p\tau + Y)} \sin(\omega t + \alpha), \quad (34')
\end{aligned}$$

where

$$\begin{aligned}
\alpha = & (-1)^y \left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{\Pi k}{n} \right) chk\tau (p-y) - (M+N) shkp\tau - \\
& - \left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) chkp\tau - (-1)^y (N-M+11) shk\tau (p-y) \\
B_2 = & M \cos \left[\omega t - (m+n)\bar{\varphi} + \frac{\pi p}{2y} \right] - N \cos \left[\omega t + (m-n)\bar{\varphi} - \frac{\pi p}{2y} \right] + \\
& + (-1)^p \sqrt{\left[\left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) chkY \right]^2 + \left[(M+N) shkY \right]^2} \frac{chk\bar{\varphi}}{shk(p\tau + Y)} \sin(\omega t + \alpha_1) + \\
& + (-1)^{p-y} \sqrt{\left[(N-M+11) shk\tau (p-y) \right]^2 + \left[(M+N) shkp\tau \right]^2} \frac{chk(p\tau + Y - \bar{\varphi})}{shk(p\tau + Y)} \sin(\omega t + \alpha_2) \quad (35')
\end{aligned}$$

$$\left[\left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{lk}{n} \right) \operatorname{ch} k \tau (p-y) \right]^2 \frac{\operatorname{ch} k (p\tau + Y - \tau)}{\operatorname{sh} k (p\tau - Y)} \sin (\omega t + a_2); \quad (35')^*$$

where

$$\begin{aligned} \operatorname{tg} a_1 &= \operatorname{th} k Y \cdot \frac{M+N}{\frac{Nk}{m-n} - \frac{Mk}{m+n}}; \\ \operatorname{tg} a_2 &= \frac{\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{lk}{n}}{\frac{Nk}{m-n} - \frac{Mk}{m+n}} \operatorname{ch} k \tau (p-y); \\ B_2 &= \Pi \sin \left(\omega t - \frac{\pi}{\tau} \tau \right) + (-1)^p \sqrt{\left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) \operatorname{ch} k Y -} \\ &\quad \left((-1)^p (N - M + \Pi) \operatorname{sh} k (Y + y\tau) \right)^2 + \left((M + N) \operatorname{sh} k Y + \right. \\ &\quad \left. + (-1)^p \left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{lk}{n} \right) \operatorname{ch} k (Y + y\tau) \right)^2 \frac{\operatorname{ch} k \tau}{\operatorname{sh} k (p\tau - Y)} \sin (\omega t + a_2); \end{aligned} \quad (36')$$

where

$$\operatorname{tg} a_3 = \frac{(M+N) \operatorname{sh} k Y + (-1)^p \left(\frac{Mk}{m+n} + \frac{Nk}{m-n} - \frac{lk}{n} \right) \operatorname{ch} k (Y + y\tau)}{\left(\frac{Nk}{m-n} - \frac{Mk}{m+n} \right) \operatorname{ch} k Y - (-1)^p (N - M + \Pi) \operatorname{sh} k (Y + y\tau)};$$

Analyzing expressions (34)-(36) it may be concluded that there is a component in the magnetic field which is stationary in space and which varies with time. The amplitude of this component has a maximum value at the junction of the first and second sections, i.e., where the winding ends.

Thus, my earlier statements [2] concerning the fact that the edge effect may be completely eliminated by the use of distributed windings of a curved stator in isolated cases proves to be not altogether correct. Despite the fact that the incident wave of magnetomotive force reduces to zero at the edge, it is practically impossible for the incident induction wave at the end of the arc of the curved stator to drop to zero when using the given type of winding. This is explained by the

* $\operatorname{tg} = \tan$, $\operatorname{th} = \tanh$, $\operatorname{cth} = \coth$

fact that the reluctance of steel is significantly less than that for air.

Figure. 2 shows induction curves for the air gap of a four-pole machine for various moments of time as calculated according to Formula (35). The stator had a winding with a linear load which reduced to zero at the ends along a two-pole separation. This same figure shows the experimental curve of amplitude values of induction in the air gap. The winding of the curved stator was made according to the schematic in Fig. 3a. The diameter of the rotor equalled 100 mm. The number of loops is shown in the schematic in Fig. 3a. The coverage angle of the curve stator was 230° . The ideal no-load current for a 120-volt 500-cps source was equal to 0.3 amps. The coefficient k was equal to 0.12 cm^{-1} . Ideal no-load conditions were obtained by replacing the regular rotor with one without a squirrel cage. The method of the experiment has been described earlier [2].

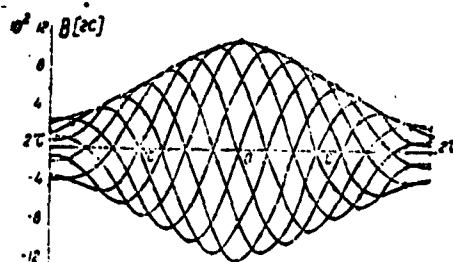


Fig. 2. Induction curves in the air gap of a four-pole induction machine with a curved stator. Curves with points - calculated, curve with stars - experimental.

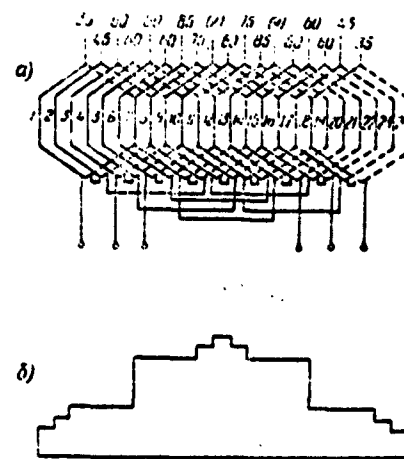


Fig. 3. a) Schematic of the distribution of the winding of a curved stator. b) Nature of the variation of linear load over the length of arc of the stator. The numbers on top indicate the number of turns in the half-coils.

From a comparison of the curves it may be concluded that the calculation formulas very accurately describe the phenomena which take place in the machine.

Some divergence of the calculated and experimental curves at the ends of the arc of the stator may be explained not only by the assumptions made in the derivation of the computational formulas, but also by the fact that variation of the linear load at the ends of the curved stator was not strictly sinusoidal (see Fig. 3b).

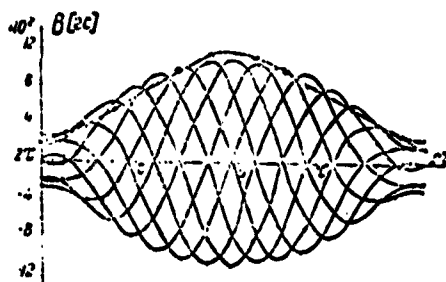


Fig. 4. Curves of the induction in the gap, without taking pulsative fields into account. Curves with points — calculation; curve with stars — experimental.

Figure 4 shows induction calculation curves without regard for the pulsating component. For comparison, the experimental curve is also shown in this figure.

Comparing the curves in Figs. 2 and 4, we see that the amplitude of the fields that differ from the basic wave is very insignificant, and these fields may be neglected in engineering calculations of machines with a curved stator having the new type of winding.

Using the described method of deriving the formula for induction in the air gap of a machine with a curved stator, having a constant linear load over the length of the arc, we obtain the following result:

$$B = \frac{\mu_0 A}{k^2 + \frac{\pi^2}{\tau^2}} \left\{ \frac{\pi}{\tau} \sin \left(\omega t - \frac{\pi}{\tau} y \right) - (-1)^p \sqrt{\left(\frac{\pi}{\tau} sh k Y \right)^2 + \left(k ch k Y \right)^2} \frac{ch k y}{sh k (p\tau + Y)} \sin (\omega t + \alpha) \right\},$$

$$\operatorname{tg} \alpha = k \frac{\pi}{\tau} ch k Y.$$

where

$$B_k = (-1)^p \frac{\mu_0 A}{k^2 + \frac{\pi^2}{\tau^2}} \sqrt{\left(\frac{\pi}{\tau} \operatorname{sh} k p \tau\right)^2 + \left(k \operatorname{ch} k p \tau\right)^2} \frac{\operatorname{ch} k (p \tau + Y - y)}{\operatorname{sh} k (p \tau + Y)} \sin (\omega t + \alpha_1),$$

$$\operatorname{tg} \alpha_1 = -k \frac{\tau}{\pi} \operatorname{cth} k p \tau,$$

where

B = induction in the air gap in the section occupied by the winding,

B_k = induction in the air gap in the section where there is no winding, and

y = present value of the coordinate along the length of the arc.

In this formula in contrast to the analogous formula [1], the present coordinate y has one reading origin at the center of the arc of the curved stator.

In addition, in the expression for induction under the active zone of the stator, there is no term which varies proportionally with the hyperbolic sine of the coordinate of the point. This hyperbolic sine, in accordance with Shturman [1], makes the value of induction dependent on the selection of the sign in the coordinate system i.e., it makes the expression asymmetric with respect to the middle of the arc; this asymmetry depends on the selection of the directions of the axes of the coordinate system. The coefficients of the other terms which determine the pulsating fields, also differ from the corresponding coefficients in Shturman's formulas [1].

Submitted March 25, 1959

REFERENCES

1. G. I. Shturman. Induktsionnyye mashiny s razomknutym magnitoprovozom. "Electrichestvo," No. 10, 1946.
2. A. A. Lebedev. Umen'sheniye poter' ot krayevogo effekta dugovogo statora. Trudy LKVVIA im. A. F. Mozhayaskovo, No. 282, 1959.

DISTRIBUTION LIST

| DEPARTMENT OF DEFENSE | Nr. Copies | MAJOR AIR COMMANDS | Nr. Copies |
|-----------------------|------------|--------------------|------------|
| | | AFSC | |
| | | SCFDD | 1 |
| | | ASTIA | 25 |
| HEADQUARTERS USAF | | TDBTL | 5 |
| | | TDBDP | 5 |
| AFCIN-3D2 | 1 | AEDC (AEY) | 1 |
| ARL (ARB) | 1 | SSD (SSF) | 2 |
| | | BSD (BSF) | 1 |
| OTHER AGENCIES | | AFTTC (FTY) | 1 |
| | | AFSWC (SWF) | 1 |
| | | ASD (ASYIM) | 1 |
| | | TDEPA (SMITH) | 1 |
| CIA | 1 | | |
| NSA | 6 | | |
| DIA | 9 | | |
| AID | 2 | | |
| OTS | 2 | | |
| AEC | 2 | | |
| PWS | 1 | | |
| NASA | 1 | | |
| ARMY | 3 | | |
| NAVY | 3 | | |
| NAFEC | 1 | | |
| RAND | 1 | | |
| POE | 12 | | |